

2540 km: Bimagic baseline for neutrino oscillation parameters

Amol Dighe,¹ Srubabati Goswami,² and Shamayita Ray³

¹*Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400005, India*

²*Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India*

³*Laboratory for Elementary-Particle Physics, Cornell University, Ithaca, NY 14853, USA*

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We show that a source-to-detector distance of 2540 km, motivated recently [1] for a narrow band superbeam, offers multiple advantages for a low energy neutrino factory with a detector that can identify muon charge. At this baseline, for any neutrino hierarchy, the wrong-sign muon signal is almost independent of CP violation and θ_{13} in certain energy ranges. This allows the identification of the hierarchy in a clean way. In addition, part of the muon spectrum is also sensitive to the CP violating phase and θ_{13} , so that the same setup can be used to probe these parameters as well.

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Introduction.— The data from ongoing neutrino experiments confirm that neutrinos have distinct masses m_1, m_2, m_3 and the three neutrino flavors ν_e, ν_μ, ν_τ mix among themselves. While the mass squared difference $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and the magnitude of $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$, as well as two of the mixing angles, θ_{12} and θ_{23} , are well measured, three parameters of the leptonic mixing matrix still remain elusive: the mixing angle θ_{13} , the sign of Δm_{31}^2 , and the CP phase δ_{CP} [2]. The determination of hierarchy (NH/normal: $\Delta m_{31}^2 > 0$, IH/inverted: $\Delta m_{31}^2 < 0$), in particular, would be crucial in identifying the mechanism of neutrino mass generation [3].

If the actual value of θ_{13} is not much below the current 3σ bound of $\theta_{13} < 12^\circ$, it may be measured at detectors at a distance of $\lesssim 1$ km from a reactor/accelerator. In order to determine the hierarchy, however, the most efficient avenue is to have the neutrinos travel through Earth for thousands of km before detection. Here, the difference between Earth matter effects in the two hierarchies can help in distinguishing them. This can be achieved, for instance, by using the decay of accelerated muons $-\mu^+$ or μ^- – as a source (“neutrino factory” (NF)) and a detector that can detect muons and identify them as right-sign (the same sign as the source) or wrong-sign.

The wrong-sign muon signal is hailed as the “golden channel” since it is sensitive to all the three parameters: θ_{13} , the sign of Δm_{31}^2 , and δ_{CP} . However, the dependence on δ_{CP} also introduces large uncertainties, making the unambiguous determination of the true parameters difficult [4, 5]. A potential way out is to have the detector at ~ 7500 km (“the magic baseline” [5, 6]) from the source, where the effect of CP violation vanishes for both hierarchies. However, this very feature makes it impossible to measure the CP phase at this baseline. Moreover, such a long baseline requires an extremely well-collimated muon source, else the flux at the detector is highly reduced.

It is therefore desirable to look for a shorter baseline that will still give a wrong-sign muon signal independent of the CP phase for one of the hierarchies, albeit only in a part of the spectrum. The remaining part of the spectrum would still be sensitive to the CP phase and can be used to detect CP violation for the same hierarchy.

In the context of a ν_μ superbeam, it was recently pointed out [1] that the baseline of 2540 km satisfies the above condition for IH at a neutrino energy of 3.3 GeV and a narrow band neutrino beam was therefore deemed desirable. In this Letter, we point out that this baseline *also* satisfies the desired condition for NH, at the energy 1.9 GeV. The two energies at which the desired condition is satisfied are termed as magic energies, and the baseline is referred to as “bimagic”. The bimagic property, first realized in this work, makes it more desirable to have a broadband beam covering the range 1–4 GeV. We use the $e-\mu$ channel in a low energy neutrino factory (LENF) with a muon energy of 5 GeV [7], as opposed to the $\mu-e$ channel used for superbeams [1]. The detection of muons is easier compared to that of electrons. Moreover with muon charge identification, NFs do not have beam contamination problems, thus enabling sensitivity to smaller θ_{13} values. Thus, the bimagic nature in conjunction with a LENS helps in an efficient identification of hierarchy, nonzero θ_{13} and CP violation, even with a single polarity of decaying muons, as we shall motivate and demonstrate in this Letter. It is remarkable that the distance 2540 km also happens to be close to the distance between Brookhaven and Homestake [8], as well as that between CERN and Pyhasalmi mine [9], which is one of the proposed sites for the LENA detector.

The bimagic baseline.— The source beam from a neutrino factory that accelerates μ^+ consists of $\bar{\nu}_\mu$ and ν_e . Charged current interactions at the detector can give muons in two ways: the original $\bar{\nu}_\mu$ that survive as $\bar{\nu}_\mu$ give μ^+ (right-sign muons) while the original ν_e that oscillate to ν_μ give μ^- (wrong-sign muons). The oscillation probability $P_{\nu_e \rightarrow \nu_\mu}$, relevant for the wrong-sign muon signal, can be written in the constant matter density approximation as [10]

$$P_{e\mu} = 4s_{13}^2 s_{23}^2 \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} + \alpha^2 \sin^2 2\theta_{12} c_{23}^2 \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2} + 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta - \delta_{\text{CP}}) \times \frac{\sin \hat{A}\Delta}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})}, \quad (1)$$

keeping terms up to second order in $\alpha \equiv \Delta m_{21}^2/\Delta m_{31}^2$ and s_{13} . Here $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$. Also,

$$\hat{A} \equiv 2\sqrt{2}G_F n_e E_\nu / \Delta m_{31}^2, \quad \Delta \equiv \Delta m_{31}^2 L / (4E_\nu), \quad (2)$$

where G_F is the Fermi constant and n_e is the electron number density. For neutrinos, the signs of \hat{A} and Δ are positive for normal hierarchy and negative for inverted hierarchy. \hat{A} picks up an extra negative sign for anti-neutrinos. The last term in Eq. (1) clearly mixes the dependence on hierarchy and δ_{CP} , leading to a degeneracy between them [4], which can be overcome if one manages to have either $\sin(\hat{A}\Delta) = 0$ or $\sin[(1 - \hat{A})\Delta] = 0$. The first condition is achieved at the magic baseline ($L \sim 7500$ km) for all E_ν and for both the hierarchies. The second condition, on the other hand, is sensitive to hierarchy. This sensitivity can be maximized if one has $\sin[(1 - \hat{A})\Delta] = 0$ for one of the hierarchies and $\sin[(1 - \hat{A})\Delta] = \pm 1$ for the other. In such a situation, only the $\mathcal{O}(\alpha^2)$ term in Eq. (1) survives for the hierarchy for which $\sin[(1 - \hat{A})\Delta] = 0$, making $P_{e\mu}$ independent of both δ_{CP} and θ_{13} . At the same time, for the other hierarchy the first term in Eq. (1) enhances the number of events as well as θ_{13} sensitivity, and the third term enhances the sensitivity to δ_{CP} .

If we demand “IH-noCP” (no sensitivity to CP phase in IH), these conditions imply

$$(1 + |\hat{A}|) \cdot |\Delta| = n\pi \quad \text{for IH}, \quad (3a)$$

$$(1 - |\hat{A}|) \cdot |\Delta| = (m - 1/2)\pi \quad \text{for NH}, \quad (3b)$$

where n, m are integers, $n > 0$. These two conditions are exactly satisfied at a particular baseline and energy, given by

$$\rho L (\text{km g/cc}) \approx (n - m + 1/2) \times 16300, \quad (4a)$$

$$E_\nu (\text{GeV}) = \frac{4 \Delta m_{31}^2 (\text{eV}^2) L (\text{km})}{5 (n + m - 1/2)}. \quad (4b)$$

Note that the relevant L is independent of any oscillation parameters. A viable solution for these set of equations (with $n = 1$ and $m = 1$) is $L \approx 2540$ km, $\rho = 3.2$ g/cc and $E_\nu \equiv E_{IH} \approx 3.3$ GeV, as was first pointed out in [1]. On the other hand, one may demand “NH-noCP” (no sensitivity to CP phase in NH), which leads to the conditions

$$(1 - |\hat{A}|) \cdot |\Delta| = n\pi \quad \text{for NH}, \quad (5a)$$

$$(1 + |\hat{A}|) \cdot |\Delta| = (m - 1/2)\pi \quad \text{for IH}, \quad (5b)$$

with n, m integers, $n \neq 0$ and $m > 0$. These lead to the same condition on L as in Eq. (4a) except for an overall negative sign, while E_ν continues to be given by Eq. (4b). These conditions are also satisfied at $L = 2540$ km (for $n = 1$ and $m = 2$) at $E_\nu \equiv E_{NH} \approx 1.9$ GeV. The magic energies E_{IH} and E_{NH} would be suitable for a neutrino factory with a parent muon energy of ~ 5 GeV.

Eqs. (4a, 4b) indicate that many combinations of n and m are possible for a given baseline. Indeed, the

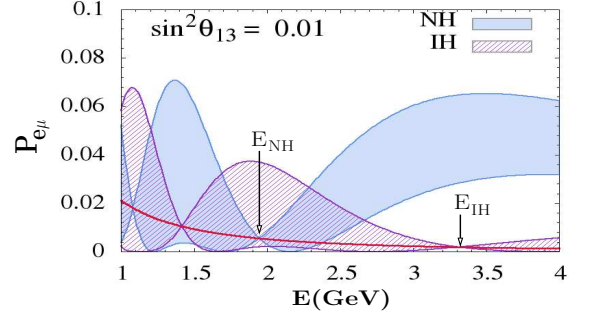


FIG. 1: Conversion probability $P_{e\mu}$ for $L = 2540$ km. The bands correspond to $\delta_{CP} \in (0, 2\pi)$. Other parameters are taken as $\Delta m_{21}^2 = 7.65 \times 10^{-5} \text{ eV}^2$, $|\Delta m_{31}^2| = 0.0024 \text{ eV}^2$, $\sin^2 \theta_{12} = 0.3$ and $\sin^2 \theta_{12} = 0.5$. The red (solid) line corresponds to $\theta_{13} = 0$.

2540 km baseline also satisfies IH-noCP at $E_{IH2} \approx 1.3$ GeV ($n = 2, m = 2$) and NH-noCP at $E_{NH2} \approx 0.9$ GeV ($n = 2, m = 3$). However the flux at these energies would be small, so we do not consider these in this Letter.

Fig. 1 shows the probability $P_{e\mu}$ for $\sin^2 \theta_{13} = 0, 0.01$. In this and all other plots, we have solved the exact neutrino propagation equation numerically using the Preliminary Reference Earth Model [11]. Clearly the IH-noCP and NH-noCP conditions are satisfied at the energies E_{IH} and E_{NH} , respectively. At E_{IH} , the probabilities $P_{e\mu}$ for NH and IH are distinct, hence a measurement of the neutrino spectrum around this energy would be a clean way of distinguishing between the hierarchies. The oscillatory nature of $P_{e\mu}$ for non-zero θ_{13} vis-a-vis the monotonic behavior for $\theta_{13} = 0$ helps in the discovery of a nonzero θ_{13} . Finally, the significant widths of the bands (near E_{IH} for NH, and near E_{NH} for IH) imply sensitivity to δ_{CP} .

The simplified forms of probabilities at the magic energies offer insights into the CP sensitivity at this baseline. At E_{IH} , we have

$$\begin{aligned} P_{e\mu}(\text{IH}) &\approx 18\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2, \\ P_{e\mu}(\text{NH}) &\approx 18\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2 + 9s_{13}^2 s_{23}^2 \\ &\quad - 18\sqrt{2}\alpha s_{12} c_{12} s_{23} c_{23} s_{13} \cos(\delta_{CP} + \pi/4), \end{aligned} \quad (6)$$

while at E_{NH} , we have

$$\begin{aligned} P_{e\mu}(\text{NH}) &\approx 50\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2, \\ P_{e\mu}(\text{IH}) &\approx 50\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2 + (25/9)s_{13}^2 s_{23}^2 \\ &\quad - (50\sqrt{2}/3)\alpha s_{12} c_{12} s_{23} c_{23} s_{13} \cos(\delta_{CP} + \pi/4). \end{aligned} \quad (7)$$

Near the magic energies, where the CP sensitivity is the highest, the δ_{CP} values giving the highest and the lowest probabilities would be $3\pi/4$ and $7\pi/4$, respectively.

Experimental setup and numerical simulation.— We use a magnetized totally active scintillator detector (TASD) which is generally used in the context of a LENF [7]. We use a 25 kt detector with a energy threshold of 1

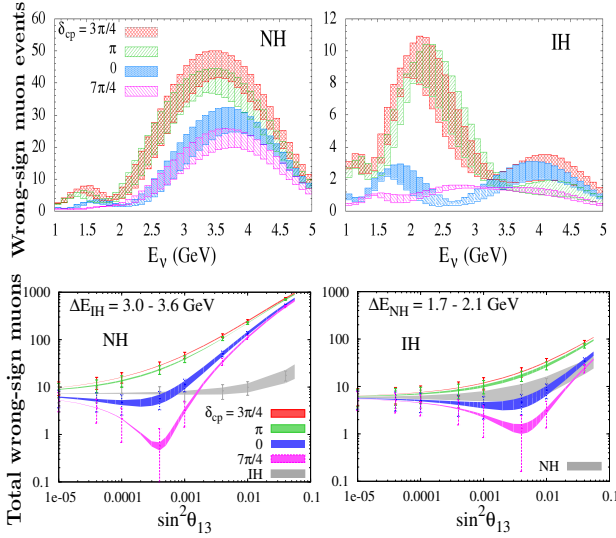


FIG. 2: Top panel: energy spectra of wrong-sign muons for NH (left) and IH(right). Bottom panel: number of events for 1 year run, in the bins ΔE_{IH} and ΔE_{NH} as a function of θ_{13} . The bands correspond to 5% error in Δm_{31}^2 . The bands in the top panels also include a 10% error around $\sin^2 \theta_{13} = 0.01$.

GeV. We choose a typical Neutrino factory setup with 5 GeV parent muon energy and 5×10^{21} useful muon decays per year [12, 14]. We consider the running with only one polarity μ^+ of the parent muon, so that we have a neutrino flux consisting of $\bar{\nu}_\mu$ and ν_e . We assume a muon detection efficiency of 94% for energies above 1 GeV, 10% energy resolution for the whole energy range up to 5 GeV and a background level of 10^{-3} for the $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ channels. Detection of ν_e or $\bar{\nu}_e$ is not considered in this study, which seems to have a very small effect when the initial flux is as large as above [14]. A 2.5% normalization error and 0.01% calibration error, both for signal and background, have also been taken into account throughout this study. The detector characteristics have been simulated by GLoBES [13].

The top panel in Fig. 2 shows the energy spectra of wrong-sign muon events. For illustration, in addition to $\delta_{CP} = 0, \pi$, we choose $\delta_{CP} = 3\pi/4, 7\pi/4$ which would give the maximum δ_{CP} dependence near the magic energies, as indicated by Eqs. (6) and (7). It is clear from this figure that there is considerable sensitivity to CP phase near $E_{IH}(E_{NH}) \approx 3.3(1.9)$ GeV for NH (IH). It may be noted from the figure that the CP sensitivity for IH is actually better at slightly higher energies than E_{NH} . This is because the ν_e spectrum at the source as well as the cross section of ν_μ at the detector are strongly increasing functions of energy around $E_\nu \sim 2$ GeV, and push the peak in the IH spectrum to higher energies.

In order to illustrate the effectiveness of the magic energies, we show in the bottom panel of Fig. 2 the total number of events in two bins near the magic energies – ΔE_{IH} (3.0–3.6 GeV) and ΔE_{NH} (1.7–2.1 GeV) – as functions of θ_{13} . Clearly, the bin ΔE_{IH} itself is enough

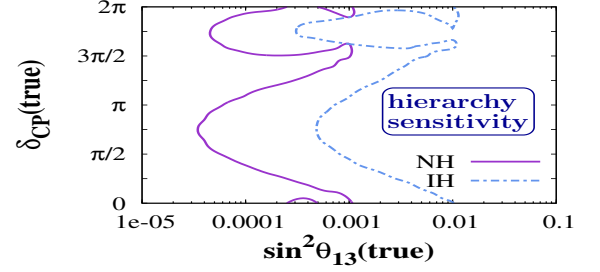


FIG. 3: The 3σ hierarchy sensitivity contours, obtained with a flux of 5×10^{21} positive muons/year on a 25 kt TASD for 2.5 years, for true hierarchies as indicated. For parameters to the right of the contours, hierarchy can be identified.

to identify the hierarchy as long as $\sin^2 \theta_{13} \gtrsim 10^{-3}$. If the actual hierarchy is NH, this bin is also sensitive to δ_{CP} . The sensitivity to θ_{13} may be estimated by comparing the error bars at different θ_{13} values. If the actual hierarchy is IH, one needs the events data from the energy bin ΔE_{NH} in order to discern δ_{CP} and θ_{13} . The actual identification of hierarchy and the measurement of δ_{CP} and θ_{13} is done by using the complete wrong-sign events spectrum as well as the right-sign events spectrum. We present below the results of this analysis.

Mass hierarchy determination.— In Fig. 3 we quantify the hierarchy sensitivity of the bimagic neutrino factory setup. The experimental data are generated with the chosen true hierarchy. The true values of $\sin^2 \theta_{13}$ and δ_{CP} are plotted along the axes while the true values of the other parameters are set to the values quoted in Fig. 1. For each pair of $\sin^2 \theta_{13}(\text{true})$ – $\delta_{CP}(\text{true})$, we obtain χ^2_{\min} by marginalizing over other parameters. We have taken 4% error on each of Δm_{21}^2 and θ_{12} , and 5% error on each of θ_{23} and Δm_{31}^2 , for calculating the priors. δ_{CP} has been varied over $(0, 2\pi)$. A 2% error has also been considered on the earth matter profile and marginalized over.

The contours in Fig. 3 suggest that if the true hierarchy is NH, then for favorable values of δ_{CP} , an exposure of $\approx 3 \times 10^{23}$ muons \times kt may determine the hierarchy at 3σ even for $\sin^2 \theta_{13}$ as small as $\sim 3 \times 10^{-5}$. If the true hierarchy is IH then that can be established at 3σ for $\sin^2 \theta_{13} \gtrsim 3 \times 10^{-4}$. This sensitivity is better than that indicated by the superbeam studies at this baseline [1, 8].

An optimized LENF setup with a baseline of 1300 km has been recently proposed [14]. However, the relatively small baseline does not allow matter effects to develop sufficiently, and one does not have the advantage of the magic energies. So the sensitivity of this setup to the hierarchy is rather limited. Indeed if the true hierarchy is NH, the bimagic baseline will rule out IH at 3σ for $\sin^2 \theta_{13}$ values almost an order of magnitude smaller than the expected reach of the 1300 km setup with the same exposure. If IH is the true hierarchy, the performance of both the setups is almost the same. Thus the bimagic baseline is a more optimal setup as far as the hierarchy

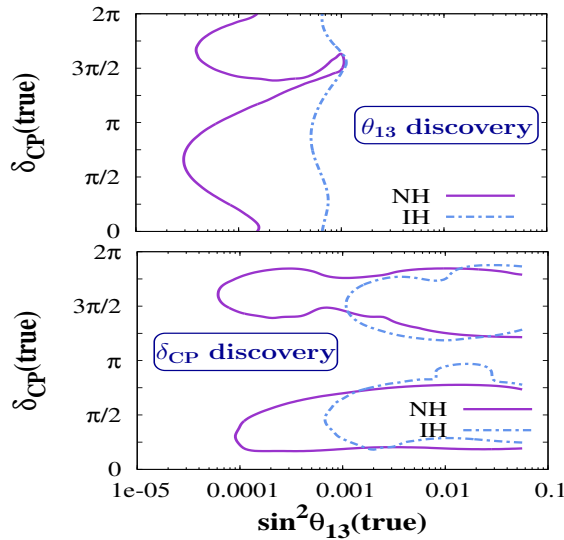


FIG. 4: The 3σ discovery contours for θ_{13} (upper panel) and CP violating phase δ_{CP} (lower panel), obtained with a flux of 5×10^{21} positive muons/year on a 25 kt TASD for 2.5 years, for true hierarchies as indicated. For parameters to the right of the contours, the discovery of the relevant parameter is possible.

is concerned.

θ_{13} and δ_{CP} measurement.— The top panel of Fig. 4 shows that the exposure of $\approx 3 \times 10^{23}$ muons \times kt will be able to discover a nonzero θ_{13} to 3σ as long as $\sin^2 \theta_{13} \gtrsim 10^{-3}$ for either hierarchy and for any δ_{CP} value. For NH and $\delta_{CP} \approx 3\pi/4$, the discovery of θ_{13} is possible even for $\sin^2 \theta_{13}$ as low as 3×10^{-5} .

The bottom panel of Fig. 4 shows the δ_{CP} discovery reach with this setup. It shows that the exposure allows the discovery of nonzero δ_{CP} for NH for $\sin^2 \theta_{13}$ as low as 10^{-4} , as long as $\delta_{CP} \approx 3\pi/4$. This is the δ_{CP} value at which we expect the highest deviation in the events

spectrum from $\delta_{CP} = 0$, as indicated by Eqs. (6) and (7). For IH, the results are about one order of magnitude worse than those for NH.

The discovery potential for θ_{13} and δ_{CP} at the bimagic baseline is comparable to that of the 1300 km setup if the true hierarchy is NH, while it is not as good if the true hierarchy is IH. However, note that this is valid if only μ^+ are available at the source. With both polarities available, the bimagic baseline would be almost as good as the 1300 km setup for θ_{13} and δ_{CP} , and will have a better sensitivity to the hierarchy. Indeed, once the hierarchy is identified – for which the bimagic baseline performs better – running the bimagic setup with μ^+ (μ^-) as the source beam for NH (IH) would offer a sensitivity similar to the 1300 km setup. Thus, overall the bimagic baseline seems like an optimal one to probe the three most important unknown parameters of the leptonic mixing matrix: θ_{13} , δ_{CP} and the sign of Δm_{31}^2 .

Conclusion.— We have shown the “bimagic” nature of the 2540 km baseline: at this baseline with judicious choice of energies, the dependence of the wrong-sign muon signal on θ_{13} and δ_{CP} can be made to vanish for either hierarchy. This energy turns out to be around 3.3 GeV for IH and 1.9 GeV for NH. This helps in an efficient identification of hierarchy even at very low θ_{13} , when one uses a neutrino factory with parent muon energy ~ 5 GeV as a source. On the other hand the sensitivity to θ_{13} and δ_{CP} is maximum at ~ 3.3 GeV for NH and 1.9 GeV for IH, allowing the determination of these parameters as well with the same beam-baseline setup. To exploit these features, a broadband beam of a neutrino factory is more effective as compared to a narrow band beam.

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